Markscheme

November 2016

Extended mathematics

On-screen examination

The markscheme may make use of the following abbreviations:

RM Assessor has the following annotations that should be used to award marks:
A0 only use to award a zero mark for an answer that has no merit e.g. awarded for the candidate that has a wrong answer with no working

NR only use when the candidate has not made any response also stamp the response with SEEN

Marks awarded by stamping the tick

SEEN Seen; must be stamped on all blank response areas and on concatenated responses

unclear

- Bullet notation means award 1 mark – see example 1 below

ECF Marks that can be awarded as **error carried forward** from previous results in the question
BOD Benefit of the doubt
MR misread
NWS no working shown
SC special case
OE or equivalent
WTTE or words to that effect or accept incomplete calculator display
AG Answer given

Example 1
- 1 mark awarded and corresponding notes are aligned

b	• Show clear line of reasoning in the method • 4	45 & 49 seen **OE** *eg*, $49 = 45 + x$ **ACCEPT** 45 + X/10 = 4.9 **and** Ans 4	**2**

Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

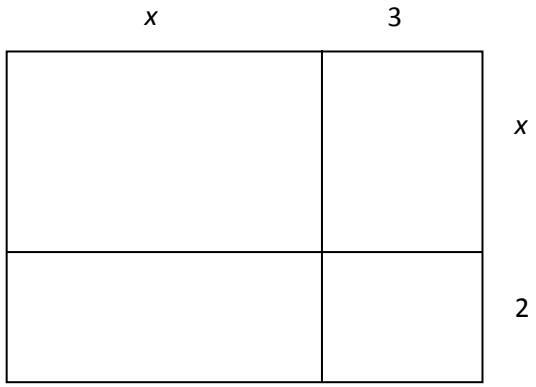
a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
b) If an answer resulting from **ECF** is inappropriate (*eg*, negative distances or sin$x$ > 1) then subsequent marks should not be awarded.
c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
d) To award **ECF** marks for a question part, **there must be working present for that part**.
e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
f) A misread (**MR**) is an error**. ECF** is normally awarded.

General points

a) As this is an international examination, accept all alternative forms of **notation,** for example 1.9 and 1,9 or 1 000 or 1.000. However **DO NOT ACCEPT** incorrect mathematical notation e.g x^2 for x^2 unless noted otherwise in the MS.
b) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer,** then the last mark cannot be awarded.
c) Where candidates have written two solutions to a question, mark the response that deserves more marks.
d) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** or equivalent e.g. $\frac{1}{2}$ or 1/2 or $1\div 2$ and $\frac{x}{2}$ $x/2$ or $x\div 2$
e) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
f) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
g) **Accept seeing equation not in-line,**
h) **Accept notation errors in intermediate steps,**
i) When a calculator screenshot is taken, accept not seeing the whole operation

Task 1

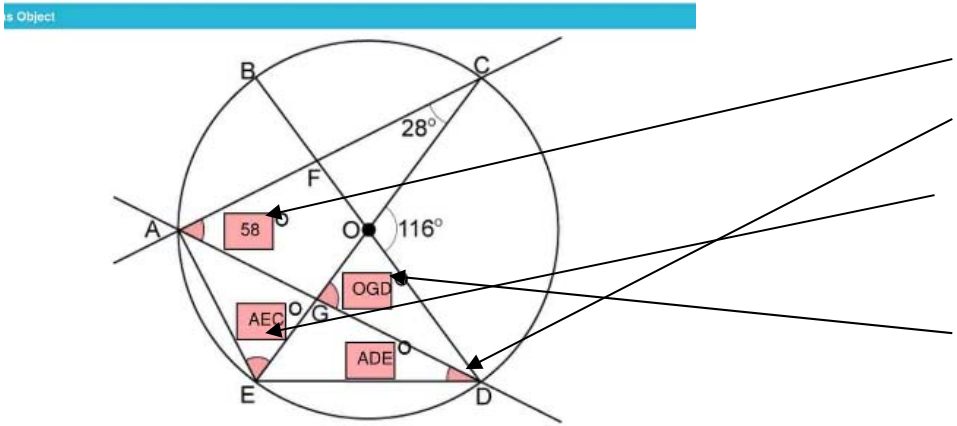
Answers			Notes	Total
1		•[1] set up of one correct equation •[2] set up another correct equation •[3] correct working elimination or substitution or trial and improvement •[4] b = 35 •[5] w = 100	•[1] $w + b = 135$ •[2] $2w + 3b = 305$ •[3] $w + 2b = 270$ or $2(135 – b) + 3b = 305$ An elimination or substitution step or evidence of trial and improvement must be seen For correct answers with evidences of trial and improvement method: Award 3 marks (marks •[3] •[4] •[5]) Marks •[1] and •[2] can still be awarded for two equations seen For correct answers with no working: Award 2 marks For •[1] and •[2] accept equations in words OE Equations seen (•[1] and •[2] awarded), then for correct answers with no further working award 4 marks in total	**5**

2	a	•[1] two correct •[2] three correct •[3] four correct 		**3**
	b	•[1] correct expression for the total area •[2] equating their expression with 210 •[3] their expression - 210 = 0 •[4] correct factorisation of their equation, or correct substitution into the quadratic formula for their equation •[5] their correctly calculated positive value of x selected	•[1] $x^2 + 3x + 2x + 6$ **OR** $x^2 + 5x + 6$ **OR** (x+3)*(x+2) •[2] $x^2 + 3x + 2x + 6 = 210$ **OR** $x^2 + 5x + 6 = 210$ •[3] $x^2 + 5x - 204 = 0$ •[4] $(x + 17)(x - 12) = 0$ •[5] $x = 12$ Footnote: The positive x value has to be selected in order to award •[5] $x = 12$ without working award 4 marks $x = 12$ with trial and error working award 4 marks $x = 12$ with incorrect working award 0 marks $x = 12$ with one correct algebraic step award 5 marks **SC:** Candidate who does not put their expression =210 and then solves $x^2 + 5x + 6 = 0$ award 2 marks	**5**

3	a	$\frac{2\times1+3\times4+4\times3+5\times6+6\times4+7\times2}{20}=\frac{94}{20}$ $\frac{2\times1+3\times4+4\times3+5\times6+6\times4+7\times2}{20}=\frac{94}{20}$ •[1] multiplying at least three grades by their frequency OR evidence of adding repeated grades •[2] adding correctly the values of grade multiplied by the frequency or adding correctly the repeated grades •[3] dividing by 20 •[4] 94/20 4.7 AG	•[1] seeing 2 x 1 and 3 x 4 and 4 x 3 for example •[2] adding 2 x 1 + 3 x 4 + 4 x 3 ..... for example •[4] do not accept any other value but 94/20 (OE) Footnote: When a calculator screenshot is taken, accept not seeing the whole fraction (WTTE) 94/20 (OE) with any evidence of correct working from the above award 4 marks 94/20 (OE) with no working award 3 marks
	b	•[1] Evidence of working to find the middle value. •[2] 5	•[1] attempt to add 1 + 4 + 3 or 2 + 4 + 6 or writing the repeated grades in order •[2] 5 with no working award 2 marks
	c	$\frac{6}{20}\times\frac{8}{20}=0.12$ •[1] 6/20 OR •[1] 8/20 •[2] multiplies by $\frac{8}{20}$ OR •[2] multiplies by 6/20 •[3] their value after multiplying fractions	•[3] $\frac{3}{25}$ **OR** $\frac{48}{400}$ **OR** 0.12 OR their values correctly multiplied $\frac{48}{400}$ or 0.12 with no working award 2 marks Footnote: All fractions must be less than 1. Adding 6/20+8/20 =14/20 award 1 mark only (for the 6/20 seen)

	d	$\frac{their\,(48/400)}{their\,(6/20)} = \frac{8}{20}$ $\frac{their\,(48/400)}{their\,(6/20)} = \frac{8}{20}$ **OR** $\frac{2}{5}$ **OR** 0.4 •[1] seeing their 6/20 •[2] dividing their (48/400) by their (6/20) •[3] the correct value after division	ECF from part (c) $\frac{8}{20}$ **OR** 0.4 Correct answer without working award 2 marks	**3**

4	a	• (DAC =) 58 (degrees)		**1**
	b	• (ADE =) 28		**1**
	c	•[1] 180 – (90 + 28) **OR** 90 – 28 •[2] (AEC =) 62	62 with no working award 2 marks	**2**
	d	•[1] (OED =) 58 •[2] (OGD =) sum of their 58 OED and their 28 ADE •[3] Their = 86 **OR** Their 58(their DAC)+28 = 86 •[1] Seeing 58 •[2] Adding 28 •[3] 86	•[1] seeing 58 •[2] (OGD =) 58 + 28 86 with no working award 2 marks	**3**



a) 58

b) 28

c) 62

d) 86 180 -(58 +28) = 180 - 86

or exterior angle = 58 + 28

5		•[1] tan22 •[2] substitutes correctly into tan ratio •[3] calculates the height above the theodolite •[4] (their) height + 1.2 •[5] rounds their height correctly to the nearest cm Alternatively •[1] Substitutes correctly into cosine ratio •[2] Calculates the length of the hypotenuse •[3] calculates the height above the theodolite (using Pythagoras) •[4] (their) height + 1.2 •[5] rounds their height correctly to the nearest cm	•[1] accept not seeing this step, can be implied by •[2] •[2] $\tan 22 = \frac{height}{57.25}$ •[3] height = 57.25 x tan 22 = 23.1305…. •[4] 23.1305…+ 1.2 = 24.3305… •[5] 24.33 (m). Accept 2433 (cm) Accept seeing rounding to nearest cm in earlier step Footnote: WTTE accept incomplete calculator display Alternatively •[1] $\cos(22) = \frac{57.25}{H}$ •[2] hypotenuse=61.746… •[3] $\sqrt{(\text{their } 61.746..)^2 - 57.25^2} = 23.1305...$ •[4] (their23.1305)+1.2 •[5] 24.33 (m). Accept 2433 (cm) Accept seeing rounding to nearest cm in earlier step 23.1305 with no working award 2 marks 23.13 with no working award 3 marks 24.33 with no working award 4 marks 24.33 with one correct step award 5 marks Footnote: WTTE accept incomplete calculator display	**5**

	b	•[1] attempt to use cosine rule (the selection of appropriate mathematics) •[2] correct substitution in cosine rule •[3] correctly calculating BC from their rule •[4] correctly approximating their value to the nearest cm	•[1] Evidence of using cosine rule even if not completely correct •[2] $BC^2 = 62.31^2 + 71.54^2 - 2 \times 62.31 \times 71.54 \times \cos 10.2$ •[3] BC = 15.036423… •[4] BC = 15.04m **OR** 1504cm Incorrect cosine rule can be awarded •[1]. •[4] can also be awarded for correct rounding. For incorrect use of Pythagoras only •[4] can be awarded. 15.04 with no working award 3 marks 15.036423 with no working award 2 marks	**4**

Task 2		Answers	Notes	Total
6	a	•1 substitutes y = 65000 into the equation •2 solving their equation correctly for x •3 their value approximated correctly to the nearest million	•1 65000 = 2110 x – 52818 •2 x = 55.8379… •3 56 million 55 with no working award 0 marks 55.8379… with no working award 1 mark 56 million with no working award 2 marks	**3**
	b	•1 substituting correctly their value in 6(a) into the percentage error formula •2 their percentage correctly calculated •3 their negative percentage written as +ve	•1 $(percentage\ \ errror\ =)\frac{their\ 56000000-68488000}{68488000}\times 100$ •2 –18 % Accept not seeing this step. And award it if they make the correct calculation for their formula •3 18 % Final answer must be positive and does not need to be rounded Footnote: Award •3 only if their calculation gives a negative value Accept evidence of substitution seen in calculator screenshot (even if incomplete due to screenshot limitations) (WTTE) -18% with no working award 1 mark 18% with no working award 2 marks	**3**
	c	•1 referring to gradient or slope or constant increase or steady increase •2 referring to gradient (or slope) being 2110 which is nearly 2000	•1 Accept: constant line steepness or proportional or rise over run DO NOT ACCEPT just referring to increase or positive relationship **SC:** comparing with a numerical example (other than the given 1000000 allows 2000 jobs) and showing that it is approximately true: Award 1 mark. The increase in passengers must be accompanied by a corresponding increase in jobs to award this SC.	**2**
	d	•1 substitutes (r =) 1.5 into the equation •2 re-arranges the equation for x or x^2 **OR** 3.548… seen •3 (x =) 4 million	•1 Accept not seeing this step. 1.5 = 2log(x) + 0.4 •2 Award 2 marks for $10^{\frac{1.5-0.4}{2}}$ **OR** 3.55 million 4 with no working award 2 marks	**3**

6 e

Aspect	1 mark	2 marks	3 marks	4 marks
IR: Identification of relevant information	One numerical factor mentioned from: Length of runway Number of jobs Economic activity Size of aircraft Number of passengers Safety considering the measured distance from houses	More than one numerical factor mentioned Length of runway Number of jobs Economic activity Size of aircraft Number of passengers Safety considering the measured distance from houses		
CM: Calculations	showing approximately correct numbers without showing calculations	Attempts to use the runway length (approximately 3.5) to find number of passengers **OR** Attempts to use the number of jobs assumed (18000) to find the number of passengers **SC**: writing approximate values with reference to tab1 and tab2	Uses correctly the runway length (approximately 3.5) to find number of passengers **OR** Uses correctly the number of jobs assumed (25000) to find the number of passengers	Uses correctly the runway length (approximately 3.5) to find number of passengers **AND** Uses correctly the number of jobs assumed (25000) to find the number of passengers
JD: Justification of degree of accuracy	Rounding used in any element (rounding to nearest million or 1 dp is accepted but not to 2dp) **OR** Referring to percentage error	Justifies **their** choice of rounding **OR** the implications of working with rounded values **OR** refers to the limitations of the data by attempting to calculate possible percentage error		

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		PD: sustainability of proposed airport	It is sustainable or it is not sustainable or a balanced discussion around sustainability supported by a relevant comment from: • **Safety**: It is in a safe area and referring to distance from houses or distance from river or both (even if they measure the distance). Environmental impact (pollution) • **Length of runway**: around 3.5 and possible increase in the future based on the map • **Room for expansion**: there is land to add more runways in the future and increase the number of jobs and/or economic activity. • **Economical expansion** outside the airport • **Facilities and services**: availability of transport networks	It is sustainable or it is not sustainable or a balanced discussion around sustainability supported by at least two relevant comments from: • **Safety**: It is in a safe area and referring to distance from houses or distance from river or both (even if they measure the distance). Environmental impact (pollution) • **Length of runway**: around 3.5 and possible increase in the future based on the map • **Room for expansion**: there is land to add more runways in the future and increase the number of jobs and/or economic activity. • **Economical expansion** outside the airport • **Facilities and services**: availability of transport networks				

mathemoeengtz0xxxx

Further guidance:

For 25000 jobs, the number of passengers is (25000+52818)/2110= 36.88 millions

From the length of runway 3.5, the number of passengers is 35.481m… and hence number of jobs 22046

7	a	•[1] substituting 65 correctly into the area formula •[2] 4225π or 13273.2289... •[3] substituting their Area and 30 correctly into the P_A formula •[4] calculating correctly their value of P_A •[5] substituting correctly their value of P_A into the P_E formula •[6] their value of P_E calculated correctly in watt •[7] writing their value of P_E to nearest kw	Allow their area using π or 3.14 or 22/7 •[1] (Area =) $\pi \times 65^2$ · Accept not seeing this step. 4225π with no working award 2 marks •[3] $P_A = 0.6$ (their area)(30^3) •[4] (=) 215026309.2 or 214917300 or 215112857.1 •[5] $P_E = 0.45$(their 215026309.2 or 214917300 or 215112857) •[6] (=) 96761839.13 or 96712785 or 96800785.71 (w) •[7] (=) 96762 or 96713 or 96801 (kw) 96762000 or 96713000 or 96801000 do not allow the •[7] mark 96761839.13 or 96712785 or 96800785.71 with no working award 5 marks 96762 or 96713 or 96801 with no working award 6 marks 96762 or 96713 or 96801 with one correct step seen award 7 marks Footnote: Substituting their area into incorrect formula does not allow •[3] mark. •[4] can be awarded as ECF only if their formula is not becoming easier (example: If they do not cube the velocity then the formula is easier and hence they are not awarded •[4]) The rest of the bullets (•[5] , •[6], •[7])can be awarded as ECF if appropriate	**7**
	b	•[1] 5/2 or 2.5 or 650 seen or 5x130 or 130/2 or 65 •[2] For multiplying 2.5 by 130 or dividing the 650 by 2 or multiplying the radius of one (130/2 or 65) by 5 325 AG	Accept showing that half way is 130+130+65=325 for 2 marks	**2**

7 c

Aspect	1 mark	2 marks
IR: Identification of strategy	One mentioned from: • Consider the length or width and the diameter of turning zone • SC not numerical: bases need to be staggered. Or accept "diagonally placed" • Fitting three rows in the 1800 instead of only two • Making turning zones closest possible • Consider total area and area of one turbine • Refer to turning zones	More than one mentioned from • Consider the length or width and the diameter of turning zone • SC not numerical: bases need to be staggered. Or accept "diagonally placed" • Fitting three rows in the 1800 instead of only two • Making turning zones closest possible • Consider total area and area of one turbine • Refer to turning zones
CM a: Calculates number of wind turbines	Number of wind turbines : 10–13 Or can be seen on the canvas 10–13 turbines fitting inside the area OR more than 13 turbines but overlapping inside the area	Number of wind turbines 14 or more Or can be seen on the canvas 14 or more turbines fitting inside the area Accept seeing zones of turbines partially outside the area (strictly speaking the centre should be inside the area)
CM b: Estimate Power Output	Attempts to substitute their area into formula of P_A and P_E	Calculates correctly their P_E = their number of turbines from their diagram x their P_E of one turbine

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7	c	PD: sustainability of wind farm	A relevant comment from: • **The diameter of the turbine:** For better use of this land maybe we need turbines with different diameter • **The average wind speed:** This area experiences an average wind speed 30 ms^{-1} and since the turbines do not operate beyond this wind speed of 30 then this means that the turbines will not be operating about half the time (at least). • **Possibility of expanding the area**: When providing an area there has to be consideration of possible extension and extra area added. OR as there is no more land to add more wind turbines so output cannot be increased above the maximum • **Turning/turbulence zones of wind turbines**: There should be no problem if the circular zone of turbines on the extremities go beyond the land because this will not affect their effectiveness but the problem is if they overlap inside the land because this will affect their effectiveness • **Maintenance or environmental impacts or general relevant points**	At least two relevant comments from: • **The diameter of the turbine:** For better use of this land maybe we need turbines with different diameter • **The average wind speed:** This area experiences an average wind speed 30 ms^{-1} and since the turbines do not operate beyond this wind speed of 30 then this means that the turbines will not be operating about half the time (at least). • **Possibility of expanding the area**: When providing an area there has to be consideration of possible extension and extra area added. OR as there is no more land to add more wind turbines so output cannot be increased above the maximum • **Turning/turbulence zones of wind turbines**: There should be no problem if the circular zone of turbines on the extremities go beyond the land because this will not affect their effectiveness but the problem is if they overlap inside the land because this will affect their effectiveness • **Maintenance or environmental impacts or general relevant points**		

7	c	JD: Justification of degree of accuracy	Rounding used in any element **OR** Simple comment like: • I used the actual answer not the rounded one from 7a) so the accuracy is exact • My answer is not accurate as I think I can fit more turbines in the area	Justifies **their** choice of rounding **OR** the implications of working with rounded values Justified comment like: • Wind speed cannot be guaranteed so the P_E is not reliable • I used rounded answer form 7a) and the power output may differ by …(and they calculate the value) if the answer was not rounded		

Task 3 (total 39 marks)

Answers			Notes	Total
8	a	•[1] relation between x-coordinates: states the *x* values of A is the same as C but negative (or opposite) •[2] relation between y-coordinates: states the *y* values of A and C are the same **OR** both 0	**SC:** Any correct comment(s) about the location award 1 mark Example: The x coordinates of both are equally apart from the y-axis OR They are both on the x-axis and equally distant from y-axis OR A and C are reflection of each other on the y-axis SC: Comment on both coordinates saying **they (or both)** are opposite signs or they are multiplied by -1 award 2 marks	**2**
	b	• the *x* coordinate of Q is half the *x* coordinate of C (or *x*-coordinate of C double *x*-coordinate of Q		**1**
	c	•[1] $(Q_4=)$ (5,3) •[2] $(Q_5=)$ (6,3) •[3] $(Q_6=)$ (7,3)		**3**
	d	•[1] one correct term •[2] fully correct $X_c = 2n + 2$	Accept equivalent expressions with any correct rearrangement or not simplified for 2 marks example: 4+2(n-1) OE award 2 marks Award 1 mark for each correct term	**2**
	e	• $X_Q = (2n + 2)/2$ or $n + 1$ OE	Allow ECF from (d)	**1**
	f	•[1] substitutes a number $n \geq 4$ into their equation from (e) •[2] compares with the corresponding value in the table for $4 \leq n \leq 6$ or compares with predictions for $n \geq 7$ •[3] acknowledges that the two values above are equal	**SC** if "tested" correctly with a value of $n \leq 3$ award 1 mark	**3**
	g	• (Midpoint =) $(y_1 + y_2)/2$ **AND** $(0 + 6)/2$	Formula must be seen	**1**

	h	$\frac{3-0}{2--4}$ •[1] for numerator 3 – 0 or 0-3 or vertical distance correctly described in words •[2] for denominator 2– –4 or -4-2 or horizontal difference correctly described in words	Accept inappropriate notation (example: 3-0/2—4)	**2**

8 i

Mark	Predictions (P)	Description (D)	Testing (T)	Verifying (V)	Justify/proof (J)	Notation and terminology (N)	Communication (L)
1	Attempted to make predictions for any set of data	Attempted to describe a pattern	Attempted to test their described pattern or general rule for n≤3	Attempted to verify their general rule for n≥4 (ex: substitutes in their formula n≥4)	Attempted to justify their described pattern or general rule	The notation and terminology have significant errors	No communication Only calculations or algebraic steps
2	Correctly predicted terms for different sets of data	Correctly described one pattern (or a simple pattern)	Tested correctly their general rule for n≤3	Calculates correctly their value for an n≥4 and mentions the corresponding value in the table	Justified the general rule for the square root or the length AQ	The notation and terminology are mostly correct Award only if D4 is achieved	Weak communication
3	Correctly predicted most of terms for all sets of data	Correctly described pattern for the square root or the length AQ		Comment comparing the values above to verify	Correctly proved the general rule for the square root		Good communication Award only if J2 is achieved
4	Correctly predicted up to n=6 for all sets of data	Attempted to describe a correct pattern as general rule for the square root or the length AQ			Correctly proved the general rule for the length AQ		
5		Correctly described pattern as general rule for the square root					
6		Correctly described pattern as general rule for the length AQ					

SC If only the set of lengths is predicted correctly and all others are empty or wrong award **(2 marks)**.

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